TRINITY COLLEGE

Mathematics Applications Units 3, 4
Test 4 2018

Calculator Assumed
Finance

STUDENT'S NAME Solns

DATE: Thursday 26 July **TIME:** 50 minutes **MARKS**: 48

INSTRUCTIONS:

Standard Items:	Pens, pencils, drawing templates, eraser
Special Items:	Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Tom invests \$8 000 into a bank account attracting simple interest at 4.5% p.a. Determine the total value of the investment after 7 years.

$I = PrT$ ✓

$= 8000 \times \frac{4.5}{100} \times 7$

$= 2520$ ✓

$\therefore$ Total $= P + I$

$= 8000 + 2520$

$= \$10,520$ ✓

2. (5 marks)

John has won some money and is looking to invest it with a bank. The bank gives John the following options:

Option A:	4.85% p.a. compounded monthly
Option B:	4.87% p.a. compounded quarterly
Option C:	4.80% p.a. compounded daily

Determine, with reasons, which option John should take if he wants to maximise his investment.

Opt A $N = 12$, $APR = 4.85$ $i_{eff} = 4.9593\%$ ✓

Opt B $N = 4$, $APR = 4.87$ $i_{eff} = 4.95966\%$ ✓

Opt C $N = 365$, $APR = 4.80$ $i_{eff} = 4.9167\%$ ✓

∴ Option B ✓ because it is the greatest effective rate ✓

3. (5 marks)

Two banks offer Reilly and Joey the following investment packages:

Bank A:	6.5% p.a. compounded monthly for six years
Bank B:	5.3% p.a. compounded annually for eight years

If Reilly invests $10 000 in Bank A, determine how much Joey needs to invest in Bank B to produce the same amount as Reilly gets from Bank A at the end of the investment period.

Reilly: $A = 10000\left(1 + \frac{6.5}{100 \times 12}\right)^{12 \times 6}$ ✓ ✓

$= 14754.27$ ✓

Joey: $14754.27 = P\left(1 + \frac{5.3}{100}\right)^{8}$ ✓

$\Rightarrow P = 9760.92$ ✓

∴ Joey needs to invest $9760.92

4. (9 marks)

To purchase a second-hand car, David decides to start saving. He currently has $800 and can contribute additional payments of $60 at the end of each month.

The table below shows the amount in the account at the start of each month, interest accrued for that month, the payment contribution and the amount in the account at the end of each month for the first six months.

Month	Amount at start of month ($)	Interest ($)	Payment ($)	Balance at end of month ($)
1	800.00	2.40	60.00	862.40
2	862.40	2.59	60.00	924.99
3	924.99	2.77	60.00	987.76
4	987.76	2.96	60.00	1050.73
5	1050.73	3.15	60.00	1113.88
6	1113.88	3.34	60.00	1177.22
7	1177.22 ✓	3.53 ✓	60.00	1240.75 ✓

(a) Calculate the annual interest rate. [2]

$\frac{2.40}{800} \times 100\% \times 12 = 3.6\%$ p.a. ✓ ✓

(b) Write a recursive formula to determine the amount in the account at the end of each month. [3]

$T_0 = 800$ ✓

$T_{n+1} = T_n + T_n \times \frac{3.6}{100 \times 12} + 60$ ✓ ✓

OR $T_{n+1} = 1.003 T_n + 60$

(c) In which month will David have saved $2 000? [1]

$T_{18} = 1952.31$

$T_{19} = 2018.16$

∴ After 19 months he will have over $2000 ✓

(d) Complete the final row in the table above. [3]

5. (15 marks)

The Kettles take out a loan of $250 000 to purchase their first house. The bank offers them an introductory rate of 3.8% p.a. for the first two years and then the rate becomes 4.5% p.a. for the remaining time of the loan. Interest is added monthly and monthly repayments of $1 600 are made.

at the end

(a) (i) Express the loan repayment process for the first two years as a recursive formula. [2]

balance at the end of each month

$T_0 = 250000$ ✓

$T_{n+1} = T_n + T_n \times \frac{3.8}{100 \times 12} - 1600$ ✓

(ii) How much do the Kettles still owe after two years? [1]

$T_{24} = 229\,876.83$ ✓

(iii) How much interest have the Kettles paid after two years? [2]

$Int = Rep - Loan\ diff$

$= 24 \times 1600$ ✓ $- (250000 - 229\,876.83)$ ✓

$= 18\,276.83$ ✓

(b) How much do the Kettles owe after 10 years? [3]

This is 8 yrs after rate change

$N = 8 \times 12$ ✓

$I\% = 4.5$

$PV = 229876.8343$ ✓

$PMT = -1600$

$P/Y = 12$

$C/Y = 12$

$FV = -144\,791.87$

∴ owe $144 791.87 ✓

(c) How long does is take the Kettles to repay the loan? [2]

$N = ?$
$I = 4.5$
$PV = 229876.8343$
$PMT = -1600$
$FV = 0$
$P/Y = 12$
$C/Y = 12$

$N = 206.75$ months

$\therefore$ 207 months + 24 (first 2 yrs) ✓

$\therefore$ takes 231 months ✓

(or 19 yrs and 4 months)

(d) Determine the amount of the final repayment. [2]

After 206 months $FV = -1197.92$

$\therefore$ final rep $= 1197.92 \times \left(1 + \frac{4.5}{100 \times 12}\right)$

$= \$1202.41$ ✓✓

OR

After 207 months $FV = 397.59$ (bank owes you)

$\therefore$ final rep $= 1600 - 397.59 = \$1202.41$

(e) Calculate the total cost of the house. [3]

Total cost = # full rep + final

$= (206 + 24) \times 1600 + 1207.41$ ✓ ✓

$= \$369\,202.41$ ✓

6. (11 marks)

(a) Jack is about to retire and is planning to take an annuity from his pension fund. He sets up the pension fund on his 65th birthday with \$800 000 and he estimates the fund can generate a growth rate of 5% per year. He plans to start withdrawing an annuity of \$70 000 starting on his following birthday.

(i) For how many years will Jack be able to receive his annuity of \$70 000? [2]

N = ?
I = 5
PV = −800 000
PMT = 70 000 ✓
FV = 0
P/Y = 1
C/12 = 1

N = 17.37

∴ he can receive a full annuity for 17 ✓ yrs.

(ii) Assuming that all conditions remain the same, explain what would happen if Jack decided to withdraw \$35 000 per year instead of \$70 000 per year. [2]

His annuity would grow in value.

$800\,000 \times \frac{5}{100} = 40\,000$ ✓

Jack is withdrawing less than the interest repayment, therefor his annuity grows. ✓

(b) Joshua sets up his pension fund on January 1 2018 with a principle of \$850 000. The fund guarantees an annual growth rate of 7.1% p.a. compounded monthly and he plans to take an annuity of \$85 000 each year on January 1, starting in 2019.

(i) Calculate the balance in the fund after the annuity is withdrawn in January 2025. [2]

$N = 7$
$I = 7.1$
$PV = -850\,000$
$PMT = 85\,000$
$FV = ?$
$P/Y = 1$
$C/Y = 12$

✓

$FV = 651\,992.3973$

$\therefore$ balance is \$651 992.40 ✓

The investment fund revised its annual interest rate to 8.3% p.a. compounded monthly on January 1 2025 guaranteed for the period January 1 2025 to January 1 2030 and Joshua continued withdrawing \$85 000 as usual.

(ii) Calculate the balance in the fund after a withdrawal is made on January 1 2030. [2]

$N = 5$
$I = 8.3$
$PV = -651\,992.40$
$PMT = 85\,000$
$FV = ?$
$P/Y = 1$
$C/Y = 12$

✓

$FV = 481\,054.27$

$\therefore$ balance is \$481 054.27 ✓

(iii) Calculate, to the nearest \$100, the maximum amount Joshua could withdraw annually, starting in 2025, without decreasing his balance. [2]

$$651\,992.40 \times \left(1 + \frac{8.3}{100}\right)^{12} - 651\,992.40$$ ✓

$\therefore$ Can withdraw \$56 200 ✓

OR

$N = 5$ $PV = -651\,992.40$ $P/Y\ 1$
$I = 8.3$ $PMT = ?$ $C/Y = 12$
$FV = 651\,992.40$

$\Rightarrow PMT = 56\,222.22$

(iv) What is the name given to the type of annuity in part (b) (iii)? [1]

Perpetuity ✓